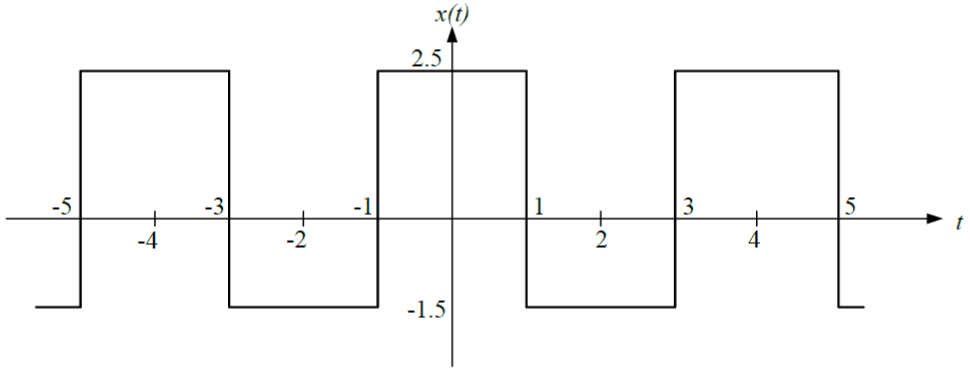
EE 362: Signals and Systems Name: Solutions

Spring 2019

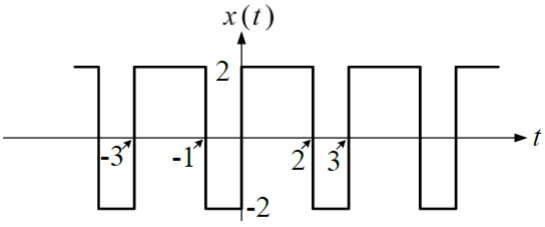
Homework #5 (Due: Thursday, February 28, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

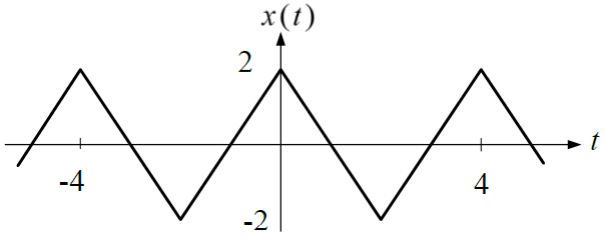
Compute the trigonometric Fourier series for each of the periodic signals shown below.



(a) [6 points]

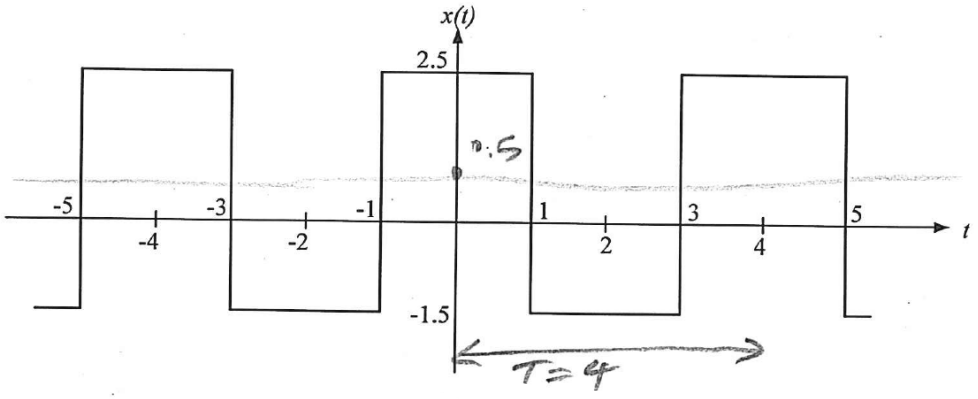


(b) [7 points]



(c) [9 points]

a)



$T = 4$; $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$; $a_0 = 0.5$; $b_k = 0$

↑ Even function

$$a_k = \frac{4}{4}\int_0^2 x(t)\cos\left(k\tfrac{\pi}{2}t\right)dt$$

$$= 2.5\int_0^1 \cos\left(k\tfrac{\pi}{2}t\right)dt - 1.5\int_1^2 \cos\left(k\tfrac{\pi}{2}t\right)dt$$

$$= \frac{2.5}{k\pi/2}\sin\left(k\tfrac{\pi}{2}t\right)\Big|_0^1 - \frac{1.5}{k\pi/2}\sin\left(k\tfrac{\pi}{2}t\right)\Big|_1^2$$

$$= \frac{5}{k\pi}\left[\sin\left(\tfrac{k\pi}{2}\right) - 0\right] - \frac{3}{k\pi}\left[\sin(k\pi) - \sin\left(\tfrac{k\pi}{2}\right)\right]$$

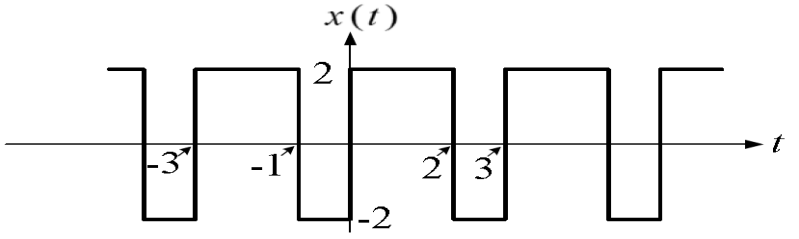
($\sin(k\pi) \to 0$)

$$= \frac{8}{k\pi}\sin\left(\tfrac{k\pi}{2}\right)$$

$$x(t) = 0.5 + \sum_{k=1}^{N} \frac{8}{k\pi}\sin\left(\tfrac{k\pi}{2}\right)\cos\left(k\tfrac{\pi}{2}t\right), \quad -\infty < t < \infty$$

b)

$T = 3 \Rightarrow \omega_0 = \frac{2\pi}{3}$



$$a_0 = \frac{1}{3}\int_0^3 x(t)\,dt = \frac{1}{3}\int_0^2 2\,dt + \frac{1}{3}\int_2^3 (-2)\,dt$$

$$= \frac{1}{3}[4-0] + \frac{1}{3}[-6+4] = \frac{2}{3}$$

$$\boxed{a_0 = \frac{2}{3}}$$

$$a_k = \frac{2}{3}\int_0^3 x(t)\cos\left(\frac{2\pi k}{3}t\right)dt$$

$$= \frac{2}{3}\int_0^2 2\cos\left(\frac{2\pi k}{3}t\right)dt + \frac{2}{3}\int_2^3 (-2)\cos\left(\frac{2\pi k}{3}t\right)dt$$

$$= \frac{4}{3}\cdot\frac{3}{2\pi k}\left[\sin\left(\frac{2\pi k}{3}t\right)\right]_{t=0}^{t=2} - \frac{4}{3}\cdot\frac{3}{2\pi k}\left[\sin\left(\frac{2\pi k}{3}\right)\right]_{t=2}^{t=3}$$

$$= \frac{2}{\pi k}\left[\sin\left(\frac{4\pi k}{3}\right) - 0\right] - \frac{2}{\pi k}\left[\sin\left(\frac{6\pi k}{3}\right) - \sin\left(\frac{4\pi k}{3}\right)\right]$$

$$= \frac{2}{\pi k}\left[2\sin\left(\frac{4\pi k}{3}\right) - \sin\left(\frac{6\pi k}{3}\right)\right]$$

$\sin\left(\frac{6\pi k}{3}\right) = 0$, for $k = 1, 2, 3, \ldots$

$$\therefore \boxed{a_k = \frac{4}{\pi k}\sin\left(\frac{4\pi k}{3}\right)}$$

$$b_k = \frac{2}{3}\int_0^3 x(t)\sin\left(\frac{2\pi k}{3}t\right)dt = \frac{2}{3}\int_0^2 2\sin\left(\frac{2\pi k}{3}t\right)dt - \frac{2}{3}\int_2^3 2\sin\left(\frac{2\pi k}{3}t\right)dt$$

$$= \frac{4}{3}\cdot\frac{3}{2\pi k}\left(\left[-\cos\left(\frac{2\pi k}{3}t\right)\right]_{t=0}^{t=2} - \left[-\cos\left(\frac{2\pi k}{3}t\right)\right]_{t=2}^{t=3}\right)$$

$$= \frac{2}{\pi k}\left(-\cos\left(\frac{4\pi k}{3}\right) + 1 + \cos\left(\frac{6\pi k}{3}\right) - \cos\left(\frac{4\pi k}{3}\right)\right)$$

$\cos\left(\frac{6\pi k}{3}\right) = 1$, for $k = 1, 2, 3, \ldots$

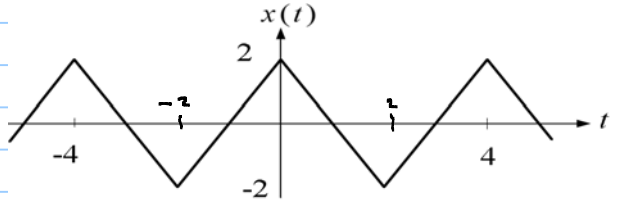
$$\therefore \boxed{b_k = \frac{4}{\pi k}\left(1 - \cos\left(\frac{4\pi k}{3}\right)\right)} \quad \text{OR}$$

$$\therefore \boxed{b_k = \frac{8}{\pi k}\sin^2\left(\frac{2\pi k}{3}\right)}$$

$$\boxed{x(t) = \frac{2}{3} + \sum_{k=1}^{\infty}\frac{4}{\pi k}\sin\left(\frac{4\pi k}{3}\right)\cos(k\omega_0 t) + \frac{8}{\pi k}\sin^2\left(\frac{2\pi k}{3}\right)\sin(k\omega_0 t)}$$

C)

$T = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$



$$a_0 = \frac{2}{4}\int_0^2 (-2t+2)\,dt$$

$$= \frac{2}{4}\left[-t^2 + 2t\right]_{t=0}^{t=2} = \frac{2}{4}\{-4+4\} = 0$$

$$\therefore \boxed{a_0 = 0}$$

Even $\Rightarrow \boxed{b_k = 0}$

$$a_k = \frac{4}{4}\int_0^2 x(t)\cos\left(\frac{\pi k}{2}t\right)dt = \int_0^2 (-2t+2)\cos\left(\frac{\pi k}{2}t\right)dt$$

let $u = (-2t+2)$ and $dv = \cos\left(\frac{\pi k}{2}t\right)dt$

$\therefore du = -2\,dt$ and $v = \frac{2}{\pi k}\sin\left(\frac{\pi k}{2}t\right)$

$\therefore \int_0^2 u\,dv = uv\Big|_0^2 - \int_0^2 v\,du$

$$\therefore a_k = \left[(2t+2)\left(\frac{2}{\pi k}\sin\left(\frac{\pi k}{2}t\right)\right)\right]_0^2 + \int_0^2 \frac{4}{\pi k}\sin\left(\frac{\pi k}{2}t\right)dt$$

$$= \frac{12}{\pi k}\sin(\pi k) - \frac{4}{\pi k}\cdot\frac{2}{\pi k}\left[\cos\left(\frac{\pi k}{2}t\right)\right]_0^2$$

$$= \frac{12}{\pi k}\sin(\pi k) - \frac{8}{(\pi k)^2}\left[\cos(\pi k) - 1\right]$$

$\sin(\pi k) = 0$, for $k = 1, 2, 3, \cdots$

$$\therefore \boxed{a_k = -\frac{8}{\pi^2 k^2}\left[\cos(\pi k) - 1\right],\ k = 1, 2, 3, \cdots} \quad \text{OR}$$

$$\therefore \boxed{a_k = \begin{cases} 0, & k \text{ is even} \\ \frac{16}{\pi^2 k^2}, & k \text{ is odd} \end{cases}} \Rightarrow \boxed{x(t) = \sum_{\substack{k=1 \\ k \text{ odd}}}^{\infty} \frac{16}{\pi^2 k^2}\cos(k\omega_0 t)}$$

and k is odd